Social Learning with Differentiated Products

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Abstract

Learning from one's friends is a key process by which consumers become informed about available products. This paper embeds social learning in a model of firms producing differentiated products. We consider how the structure of social relationships between consumers influence pricing and welfare. The model is very tractable and allows us to consider how a variety of characteristics of the social network - distribution of friendships, homophily, clustering, and correlations between an individual's preferences and number of friends - influence these outcomes. It also serves to highlight the challenges one faces in using metrics such as consumer awareness and the sensitivity of demand to prices as measures of informational efficiency in markets.

1 Introduction

An important aspect of competition in differentiated product markets is how informed consumers are about their available choices. Furthermore, a lack of information by consumers is commonly identified as an important source of inefficiency in markets, and information provision policies are often used to ameliorate it.[1] Learning from one's friends is often one of the most significant sources of information for consumers in making purchase decisions.[2] The objective of this paper is to study

[1]Examples of policies include agricultural outreach programs, energy auditing, and financial planning services. See also academic studies such as Miller and Mobarak (2015) and BenYishay and Mobarak (2015) that attempt to leverage information provision through social networks.

[2]Word of mouth has been shown to affect purchasing behavior in restaurant choices (Luca 2011), book sales (Chevalier and Mayzlin 2006), banking (Keaveney 1995), entertainment (Chintagunta et al. 2010), technological products (Herr, Kardes, and Kim 1991), and appliances and clothing (Richins 1983). These studies are also consistent with recent industry research; for example, according to the Word of Mouth Marketing Association (2011), 54% of purchase decisions are influenced by word of mouth. Also, "Word of mouth is the primary factor behind 20 to 50 percent of all purchase decisions" (Bughin et al. 2010), and "word of mouth remains the biggest influence in people's electronics (43.7%) and apparel (33.6%) purchases," National Retail Federation (2009).

the role that social learning, through a social network, plays in determining the nature of competition between firms and the welfare of participants in a differentiated product market.[3]

We develop a model of differentiated Bertrand competition, on a circle, where awareness about each firm's product diffuses through friendships. An individual becomes aware of a product if a friend has previously purchased it. Our baseline model with two firms is very tractable, allowing one to derive analytically solutions for how a wide range of network characteristics affect pricing and welfare. The results also highlight challenges for identifying more or less informationally efficient markets using surveys of consumers' awareness of products or estimates of the sensitivity of demand to the own or cross price. These metrics may move in a counterintuitive manner when the social network is changed. For instance, consumers may be aware of fewer products on average, yet the market may be more efficient.

In the baseline model with two firms, consumers are uniformly located around a circle and the firms are located opposite one another. A unit mass of consumers must choose a product to purchase. We assume that the individuals learn about the available products from a second mass of individuals who have previously purchased one or other of the products. A product is in a consumer's choice set if they have a friend who purchased that product. The intensity of price competition is determined by the prevalence of consumers who are aware of both products and may therefore respond to a change in price by either firm. In a symmetric equilibrium, these will be individuals who are located equidistant from either firm and find out about the existence of both firms from their friends. The social network influences price competition through its affect on the probability that a marginal consumer will find out about both products. In our model, inefficiencies arise due to individuals not buying from the firm located closest to themselves. The welfare loss from buying from the "wrong" firm is largest for individuals located close to one of the firms. Hence, the social network influences welfare through the likelihood that individuals get information about the firm that is closest to their location. Our analysis finds the characteristics of social networks that influence prices and welfare through these channels.

We find that increasing the number of friendships reduces prices and improves welfare, while a mean preserving spread in the distribution of friendships will increase prices and reduce welfare. When individuals are more likely to be friends with people who have similar preferences for the products then, in the two-firm case, prices are unaffected but welfare is improved; however, once we move to three firms, the change in prices is ambiguous but welfare is once again improved. We consider two different ways in which correlation between a consumer's preferences

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[3]Grossman and Shapiro (1984), in their influential model of advertising and competition in a differentiated product market, observe that "absence from our model of search, word-of-mouth and experience as sources of information is an important omission." One of the contributions of this paper is to incorporate and analyze word-of-mouth in a differentiated product market.

for the products and their number of friends may influence prices and welfare. First, we show that if marginal consumers are more likely to have more friends, then prices will decrease but welfare will be lower. Second, we show that when consumers who are located closer to one of the firms are more likely to have more friends, then that firm will charge a higher price and capture a larger share of the market. We show that welfare will also be lower than in the case where there is no correlation because of the asymmetric pricing outcome, which leads some consumers to purchase from the firm that charges a lower price but is located further away. Finally, as the number of firms is increased, prices and profits decrease; however, in the limit of a large number of firms prices are bounded away from marginal costs.

We also consider how two common measures of how well informed consumers are, average awareness of products and sensitivity of demand to price, are related to welfare. Intuitively these measures will be positively correlated with the efficiency of consumer decision making. However we find that this need not be the case when comparing two markets. When comparing across markets with different levels of homophily, the market with lower consumer awareness will be the more efficient market. Also, when comparing two markets with different correlations between an individual's number of friends and preference for the products, a market where demand is more sensitive to prices may be less efficient than a market where demand is less sensitive. These results highlight some of the challenges of using these simple metrics to compare markets without more detailed information about the network structure that facilitates learning.

2   Related Literature

We believe that this paper is the first to characterize how features of a social network influence welfare and price competition in a differentiated products market. The paper is related broadly to a large body of literature in industrial organization which studies settings where consumers are less than fully informed about the available products and/or prices. The literature has considered a variety of ways that firms provide or hide information and consumers gain access to information. On the supply side, a large literature considers the incentives of firms to undertake costly advertising (see Bagwell (2007) for an excellent review of this literature).[4] On the demand side, a significant literature has focused on the incentives for consumers to undertake costly searches to learn about the products/prices themselves (for instance, Stahl (1989) and Wolinsky (1986)).[5]   The effect of social learning

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[4]Other actions that firms strategically use include obfuscating information (Ellison and Ellison (2009), Ellison and Wolitzky (2012)), limiting comparability (Piccone and Spiegler), utilizing framing effects (Spiegler (2014)), and changing prices over time as consumers learn through experience (Bergemann and Välimäki (2006)).

[5]Other types of consumer behavior that have received attention include how stochastic dynamics of consumer switching influence competition (Sutton (1980)), naivete (Heidhues and Koszegi (2014), overconfidence (Grubb (2009)) and how rules of thumb and behavioral biases influence

on competition has received far less attention. We will discuss briefly some of the closest papers which model (or can be interpreted as models of) social learning by consumers.

In the context of more than a single firm, Galeotti (2010) develops a model of consumer search where consumers choose between searching directly for a product and searching amongst their friends, who may have searched directly themselves. The author shows how equilibrium pricing and welfare is determined by the relative costs of searching via each method. Bergemann and Välimäki (2006) study dynamic pricing of an experience good. In their context, social learning leads consumers to be less willing to experiment but for a firm to be more willing to subsidize experimentation through a lower initial price. Smallwood and Conlisk (1979) study how market shares influence quality provision and the long-run adoption by a population of consumers who use rules of thumb for selecting a new product after a breakdown of their previously preferred product. As they note, some rules of thumb correspond to sampling the population and mimicking the behavior of others. Our model and these other models share the characteristic that individuals (at least in part) receive information about their available choices from other consumers. However, our analysis is distinct from these others in considering how the characteristics of the social network, which facilitates this transmission of information, affect pricing and welfare. Goyal and Kearns (2014) consider competition between two players/firms to seed a network prior to a subsequent diffusion. Their focus is on how the dynamics of diffusion affect the inefficiency of resource use in equilibrium and how it may amplify initial differences in the budgets of the firms. It shares the similarity with the current paper that two entities are competing in the presence of a diffusion; however, our focus on differentiated Bertrand competition and the inefficiencies that arise from choosing the "wrong" product, is distinct.

Aside from models of social learning, a closely related paper is Grossman and Shapiro (1984), which considers the price and advertising equilibrium of firms located on a circle. Their model shares the similarity that consumers are less than fully informed about the available choices and the non-local nature of competition between firms. Their focus is on how informative advertising affects pricing and welfare in this context, whereas our focus is on how a social learning mechanism influences these quantities.

3 Model

There are two firms selling a horizontally differentiated product. There is a mass 1 of consumers uniformly distributed on a circle (circumference 2) where we denote a consumer's position on the circle by $y \in [-1, 1]$ and the shortest distance to the

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price competition (Spiegler (2006)) and product quality (Smallwood and Conlisk (1979)).

representative firm (w.l.o.g. let this be firm 1 located at $y = 0$) by $x = |y|$.[6] The firms are located opposite each other on the circle. We assume that a consumer receives utility $V - tx$ from purchasing the product from the representative firm and $V - t(1 - x)$ from purchasing the product from the other firm.

Consumers are connected, through friendships, to a unit mass of individuals also uniformly distributed around the circle. These individuals have previously purchased one of the products. We describe the set of social connections between the consumers who have yet to purchase and the individuals who already have, by a distribution $\{p_k\}$ where a fraction p_k of the consumers who have yet to purchase have k friends. Consumers are initially unaware of both products. Consumers become aware of one or both of the products through their friends. A consumer finds out about a product if one of their friends purchased that product. Firms compete in prices and we assume that the marginal cost to produce the good is 0.

We assume that the individuals who have already purchased a product are more likely to have purchased from their preferred firm (the firm located the shortest distance from them on the circle) and this is the same for both firms. We denote this probability by $\psi > \frac{1}{2}$, hence the probability of buying from firm 1 for the individuals for whom $x < \frac{1}{2}$ is ψ and $1 - \psi$ for $x > \frac{1}{2}$.[7]

4   Analysis

We first develop a preliminary result describing the symmetric equilibrium price P^* as a function of the mass of consumers ϕ who know about both firms; the location of these individuals as described by a p.d.f. $f(x)$ and the transportation cost t. Also in any equilibrium, let $g^*(x)$ be the probability a person at x purchases from firm 1. We note that a pricing equilibrium in pure strategies may not exist for any choice of our primitives V, t, ψ, and $\{p_k\}$. Also for some parameters, the equilibrium may be such that some consumers do not purchase from either firm. We refer the reader to the appendix, where we show that there are parameters such that a pure strategy equilibrium exists and all consumers purchase a product in equilibrium. We proceed under these conditions. We now present the following result, which finds the symmetric equilibrium price level as a function of the density of marginal consumers who know about both products and the level of differentiaton and welfare as a function of the pattern of purchases in the population.

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[6] In the two-firm case, one could equivalently locate each at either end of a line. However, in the extensions to the model where there are more than two firms, it will be more natural to locate the firms symmetrically around a circle.

[7] It is relatively straightforward to generate this as a consequence of symmetric price competition in an earlier period. For instance, if individuals are equally likely to be aware of either firm and at least some individuals are aware of both firms, $\phi > 0$. Then the individuals who are aware of both firms, in a symmetric pricing equilibrium, will choose to purchase from their most preferred firm. In this case, the fraction of individuals who purchase from their most preferred firm will be $\psi = \frac{1+\phi}{2}$.

Proposition 1 *The symmetric equilibrium price is*

$$P^* = \frac{t}{\phi f \left(\frac{1}{2}\right)}$$

and total welfare W^*

$$W^* = V - t \left[\int_0^1 (2x - 1) \, g^*(x) \, dx + \frac{1}{2} \right]$$

Proof. All proofs are contained in the appendix. ∎

Welfare is split between producers' surplus P^* and consumer surplus $W - P^*$. We note that the only inefficiency in the model is the welfare loss associated with an individual purchasing a product that is not from the firm located closest to themselves. Of course, the set of individuals who know about both products and the equilibrium pattern of purchases in the population are endogenous quantities which are influenced by the social network. The focus of our analysis is on how the social network influences these quantities and then, through the relationships described in the proposition, to relate these changes to prices and welfare.

We also consider how two metrics, the sensitivity of demand and consumer awareness, are related to welfare. Our consumer awareness metric is the fraction ϕ of individuals who are aware of both firms. A survey of (randomly chosen) consumers would find that this fraction of individuals are aware of both products. Our metric of demand sensitivity is the magnitude of the derivative of a firm's demand with respect to either the own price or the cross price. One may reasonably expect that this quantity is knowable in the region of the pricing equilibrium. This could be found by the firm at relatively small cost through experimentation, since losses due to small deviations from the profit maximizing level are second order. On the other hand it may also be knowable to an outside econometrician in the presence of small idiosyncratic cost shocks. However, estimating this quantity globally for all possible price pairs would require a great deal more variation in prices. In the region of the symmetric equilibrium, the magnitude of the sensitivity of demand for a firm to its own price and the other firm's price is given by:

$$\left| \frac{\partial Q_i}{\partial P_i} (P^*, P^*) \right| = \left| \frac{\partial Q_i}{\partial P_{-i}} (P^*, P^*) \right| = \frac{\phi f \left(\frac{1}{2}\right)}{2t} \tag{1}$$

In our model, more information cannot make an individual worse off; similarly, if an individual responds to a price change, then the individual is aware of both products and is making an efficient choice. A naive interpretation of the aforementioned metrics may then conclude that settings where consumer awareness is greater and/or demand is more responsive to price changes are environments where consumer choices are more efficient. One of the questions we answer in this paper is whether this approach is valid when changes in these metrics arise from different properties of the social network driving the diffusion of information.

4.1 Random Connections

Our first set of results concerns how changes in the social network $\{p_k\}$ affect prices and welfare. Empirically, Leskovec, Adamic and Huberman (2007) and Keller, Fay and Berry (2007) find that the average and dispersion in the amount of WOM vary greatly across product categories. An area of interest is then how the average and dispersion in the number of friends in the social network affect prices and welfare, and also whether these changes generate a positive correlation between our metrics of efficiency and welfare.

We assume that the friends of a consumer are randomly drawn. Importantly, this means that, independent of an individual's location x, a friend is equally likely to have purchased either firm's product. Under this assumption, the mass of customers who know about both products is:

$$\phi = 1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1} \tag{2}$$

and only about each firm's product:

$$\frac{1-\phi}{2} = \sum p_k \left(\frac{1}{2}\right)^{k} \tag{3}$$

for both firms 1 and 2. Finally, given our assumption of random connections, the probability of being aware of both products is independent of an individual's location and so $f(x) = 1$ for all x. Hence, the symmetric equilbrium prices in period 2 as a function of the social network are

$$P^* = \frac{t}{1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}} \tag{4}$$

The probability that an individual at a distance x purchases from the representative firm is

$$g^*(x) = \begin{cases} \frac{1+\phi}{2} \text{ for } x < \frac{1}{2} \\ \frac{1-\phi}{2} \text{ for } x > \frac{1}{2} \end{cases} \tag{5}$$

We now present the comparative statics of changes to the social network on the outcome.

Proposition 2 *Consider two distributions $\{p_k'\}, \{p_k''\}$, then prices are lower and welfare is higher under $\{p_k'\}$ if $\{p_k'\}$ FOSD $\{p_k''\}$ or if $\{p_k''\}$ is a mean preserving of $\{p_k'\}$.*

When the social network has more connections, it is more likely that individuals find out about both products. This makes each firm's demand more elastic to its price (resulting in lower prices) and reduces the number of consumers who choose

the "wrong" product. The probability of being aware of both products is a concave function in the number of friends; hence, a mean preserving spread reduces the sensitivity of a firm's demand to its price and leads to higher prices in equilibrium. In the case of random connections, welfare is determined by the fraction of consumers who know about one firm. These individuals are uniformly distributed across the population, so half will purchase from the "wrong" firm. Hence, welfare is higher when there are more elastic consumers and prices are lower.

Proposition 3 *Consider two distributions $\{p'_k\}, \{p''_k\}$ then greater awareness or more sensitive demand under one distribution than the other implies higher welfare.*

We find that changes to the distribution of friendships generate a positive relationship between the two metrics and welfare. Thus, in settings where differences in the distribution of friendships within a population are responsible for different outcomes, our metrics will be positively correlated with welfare. Here the intuitive relationship between our metrics and welfare exists, and their naive use will correctly distinguish between settings that are more or less informationally efficient.

5 Homophily

A commonly observed characteristic of social networks is the propensity for individuals to be friends with people who are similar to themselves.[8] In this section, we consider how the propensity for individuals to be friends with people who are located closer to them in product space affects prices and welfare. We find that this tendency improves welfare. In the two-firm case, homophily has no effect on prices; however in an extension to three firms, the effect is ambiguous. Increased levels of homophily have two effects on the distribution of information in the population. First, it tends to restrict the variety of information that an individual receives, leading to a small population of consumers who are aware of both products. Second, individuals located close by in product space will, on average, tend to purchase the product which is closest to their location. This means that the information they pass on is most useful for people who are similar to themselves.

We will assume that an individual located at x draws their friends from a uniform p.d.f. over individuals located within a distance $\delta < \frac{1}{2}$ of themselves with probability α and draws their friends uniformly from the population with probability $1 - \alpha$. Here the parameter α increases the degree of homophily and δ decreases the degree of

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[8] See Mcpherson, Smith-Lovin and Cook (2001): "Similarity breeds connection. This principle— the homophily principle—structures network ties of every type, including marriage, friendship, work, advice, support, information transfer, exchange, comembership, and other types of relationship. The result is that people's personal networks are homogeneous with regard to many sociodemographic, behavioral, and intrapersonal characteristics. Homophily limits people's social worlds in a way that has powerful implications for the information they receive, the attitudes they form, and the interactions they experience."

homophily. The following proposition describes the effect of the parameter α on prices and welfare.

Proposition 4 *As the amount of homophily α increases, prices and producer profits are constant; consumer and total welfare are increasing; and the fraction of the population that know about both firms decreases.*

Homophily has no effect on prices and producer profits. The reason is that introducing homophily does not change the density of consumers, who are aware of both products, at the location of the marginal consumer $x = \frac{1}{2}$. Under our assumption that the friends who have already purchased a product are more likely to buy from the firm located closest to them. A consumer located closer to firm one is more likely to have friends that purchase from firm one, and a consumer located close to firm two is more likely to have friends that purchase from firm two. However, the marginal consumers are equally likely to have friends who purchased from either firm, as they do when connections are purely random. Hence, homophily does not change the marginal consumer's propensity to know about both firms, and therefore has no effect on the prices that firms charge in equilibrium. Homophily does improve welfare because it changes the propensity of individuals located closer to either of the firms to learn about that firm from their friends. This reduces the fraction of individuals in the population who purchase from the firm located further away from themselves, thereby improving welfare.

Proposition 5 *Consider two networks with the same distribution of friendships $\{p_k\}$ but differing levels of homophily $\alpha' > \alpha''$ then, in the network with greater homophily, welfare is greater but awareness is lower and the sensitivity of demand is the same.*

The result highlights a challenge of using product awareness surveys for identifying markets which are more or less informationally efficient. As one increases the amount of homophily, consumer choices and welfare improve, but the fraction of the population which is aware of both firms decreases. Thus, a survey of consumers' awareness of products would reveal that individuals are on average less aware of the available products, despite consumers making better decisions. Here, the naive use of consumer awareness will lead one to draw the wrong conclusion.

We believe that this is the first paper to highlight the beneficial role of homophily for information diffusion in product markets.[9] The key insight is that homophily ensures that people are more likely to find out the "right" information for making their choice, as opposed to more information. The key to this property is that individuals who are similar to oneself are more likely to purchase from the firm which

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[9]Jackson and Golub show that homophily slows the rate of learning in a population. Galeotti and Mattozi highlight how homophily amongst voters may lead politicians to choose more extreme platform positions.

is the closer of the two. Hence, homophily improves the relevance of the information that an individual receives by biasing an individual's information acquisition, from their friends, towards information that is more valuable.

5.1   Example of homophily with 3 firms

It should be relatively straightforward to see that the welfare-improving effect of homophily extends readily to the case with many firms. Earlier we saw that prices were independent of the level of homophily in the two-firm case. This is no longer true when there are three firms. Here we consider an example with three firms, where increasing the amount of homophily may increase or decrease the equilibrium prices. However, the independence of prices and sensitivity of demand to homophily is a property which is specific to the case of two firms. We present an example with 3 firms to illustrate that homophily may increase or decrease prices (make demand less or more sensitive to prices).

In a similar way to before, we assume that an individual located at x draws their friends from a uniform p.d.f. over individuals located within a distance $\delta < \frac{1}{3}$ of themselves with probability α and draws their friends uniformly from the population with probability $1 - \alpha$. Once more, our focus is on the parameter α that increases the degree of homophily.

We will assume that a symmetric equilibrium is played in the first period and the likelihood that a consumer located a distance x purchases from the m'th preferred firm is given by ψ_m, where ψ_m is decreasing in m and $\sum_m \psi_m = 1$. We will consider two different cases for ψ_m. In one case, we assume that $\psi_1 = \frac{5}{9}, \psi_2 = \frac{3}{9}$ and $\psi_3 = \frac{1}{9}$. In the other case, we assume simply that $\psi_1 = 1$ and $\psi_2 = \psi_3 = 0$.

Proposition 6 *When $\psi_1 = \frac{5}{9}, \psi_2 = \frac{3}{9}$ and $\psi_3 = \frac{1}{9}$ second period prices are decreasing in the amount of homophily, and when $\psi_1 = 1$ and $\psi_2 = \psi_3 = 0$ second period prices are increasing in the amount of homophily.*

Here we see that the influence of homophily on prices is ambiguous, in that the relative likelihood of individuals purchasing from their mth preferred firm determines whether or not prices increase or decrease as homophily is increased. When there are more than two firms, homophily has two competing effects on the density of marginal consumers, those who are located equidistant from two firms. In the presence of more than two firms, there are consumers who would strictly prefer one firm's product but are located equidistant from the other two firms. These consumers may be nonetheless marginal if they are unaware of their most preferred firm. Homophily reduces the mass of these individuals because it increases the probability that consumers at this location find out about their most prefered firm. On the other hand homophily increases the probability that individuals who are indifferent between their two most preferred firms find out about both, since it reduces the probability of finding out about their least preferred firm. The former

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effect is the strongest in the case where $\psi_1 = 1$ and $\psi_2 = \psi_3 = 0$. Whereas the second effect is greater when $\psi_1 = \frac{5}{9}, \psi_2 = \frac{3}{9}$ and $\psi_3 = \frac{1}{9}$.

5.2 Clustering

A commonly observed characteristic of social networks is the prevalence of clusters of individuals who are all friends with one another. An early study of this is Rapoport (1948). More recently, Watts and Strogatz (1998) have drawn attention to this characteristic of social networks. The simplest example is a triad where three individuals are all friends. In this section, we introduce clustering into the model to contrast the connectivity effects of clustering to those of homophily. One often considers the prevalence of shared friends and cliques in a population to be reflective of a homophily-driven friendship selection process whereby individuals with similar preferences/interests/characteristics self-select into clubs, groups and socializing environments where they meet one another. Here we draw out the distinction between the competitive effect of short closed loops of friendships in a network compared to that generated by homophily. We show that short closed loops tend to reduce competitive forces because these links have a higher probability of transmitting redundant information. Unlike homophily, clustering does not improve the type of information an individual receives. Our metrics of efficiency are positively correlated with efficiency when the amount of clustering increases.

We introduce a relationship between individuals born in period 2. We assume that each individual has one friend amongst the individuals born in period 2. We assume that each period 2 individual first observes the decisions taken by their friends in period 1 and then communicates their current preferred product to their friend in period 2. After exchanging information, consumers in period 2 make their purchase decision. This may be different from the information they communicated if their friend from period 2 informs them about a product which they prefer. We consider two settings, one where the friend is selected uniformly from the population, such that the probability of an individual forming a triad is infinitesimal, and another setting where the individual shares a friend in common from the first period.

In a setting without clustering, the probability of an individual from period 2 communicating that they prefer to purchase from one or the other of the firms is independent of the information of an individual. Hence, it is equivalent to our earlier analysis in section 4.1:

$$\phi^{NC} = 1 - \sum_k p_k \left(\frac{1}{2}\right)^k \tag{6}$$

In the presence of clustering, the probability that an individual communicates that they prefer to purchase from one or another of the firms is not independent of the information of the friend. The probability that an individual only learns about a given firm from their friends in period 1 is $\left(\frac{1}{2}\right)^k$. The probability that this is the preferred firm for their friend in period 2 is $\left(\frac{1}{2}\right)^{k-1} + \frac{1}{2}\left(1 - \left(\frac{1}{2}\right)^{k-1}\right)$ where the first

term is the probability that the $k-1$ individuals who are different friends from period 1 also purchase from the same firm and $\frac{1}{2}\left(1 - \left(\frac{1}{2}\right)^{k-1}\right)$ is the probability that, in the event this is not true, the individual prefers this firm. Hence the probability of knowing about both products is:

$$\phi^C = 1 - \sum_k p_k \left(\frac{1}{2}\right)^k \left(1 + \left(\frac{1}{2}\right)^k\right) \tag{7}$$

Proposition 7 *Prices are higher and welfare, awareness and demand sensitivity are lower in a network with clustering compared to an otherwise identical network without clustering.*

These results follow from two observations. First, in a network with clustering, the fraction of individuals who know about both products (equation 7) is less than the fraction in a network absent clustering (equation 6). Second, for an individual, the location and number of their friends are independent of their own location. This results in the location of individuals who know about both firms being uniformly distributed across locations. These two observations imply that prices are given by $P = \frac{t}{\phi}$ and hence $P^{NC} < P^C$. The likelihood of purchasing from one's most preferred firm increases in ϕ, hence welfare is increasing in ϕ and welfare under clustering is lower than no clustering, $W^{NC} > W^C$. Finally, our metrics of efficiency, awareness ϕ and the demand sensitivity $\frac{\phi}{t}$ move in a way that is positively correlated with welfare, so a naive interpretation of these metrics will draw the correct conclusion regarding welfare.

We can contrast these results with those of the homophily section. One of the reasons to contrast these results is that some processes of network formation may plausibly generate homophily and clustering concurrently. For instance, if individuals form friendships by meeting in small groups based on mutual interests, then we may expect a high propensity for clustering (because the groups are small) and homophily (because individuals self-select into groups based on interests). Our results show that this process will have competing effects on welfare, and it is unclear how our metrics relate to welfare in this setting without understanding the network formation process in greater detail.

6 Correlated connectedness and valuation

For some product categories, a consumer's preferences for the products and their number of friends may be correlated. For instance, individuals who are predominantly classical music listeners or predominantly country music listeners may have more or less friends on average than individuals who are similarly interested in both genres. Alternatively, individuals located closer to one firm may have more friends than those located closer to a competing firm. For example, individuals who have a

strong preference for the graphic design characteristics of a product may have more or less friends than individuals with weak preferences for this characteristic. In this section, we consider both of these types of correlation; individuals located closer to one firm have more friends than individuals located a similar distance from the other firm, and individuals who are a similar distance from both firms have more or less friends than indiviuduals located close to one or the other of the firms.

We note that the mass of consumers ϕ in the population, who are aware of both products, is independent of any correlation between an individual's location and their number of friends. The distribution of locations within the population depends on any correlation between number of friends and location since:

$$f(x) = \frac{1 - \sum_k p_k(x)\left(\frac{1}{2}\right)^{k-1}}{\phi} \tag{8}$$

We note that if consumers at a given location have more friends, holding the population distribution constant, this increases the density at this point. For our purposes, it is useful to describe any correlation between location and number of friendships through its influence on the distribution $f(x)$.

6.1 Symmetric Correlation

In this section, we show that, as marginal consumers have more friends at the expense of infra-marginal consumers, then prices, the producer's profits and over-all welfare decrease. The effect on consumers is ambiguous, and there is a trade-off between increasing price competition through marginal consumers being better connected and increasing the efficiency of matching consumers to products.

In this section, we contrast two distributions f' and f''. We place the following assumption on these two distributions:

Assumption 1 f' and f'' are symmetric about $x = \frac{1}{2}$ and $f'(x)$ first order stochastically dominates $f''(x)$ for $x \in \left[0, \frac{1}{2}\right]$.

We continue with the understanding that increasing the connectivity of individuals closer to $\frac{1}{2}$ and decreasing the connectivity of individuals further away induces a distribution such as f'' which is FOSD by f'. Thus, a distribution such as f' exhibits greater correlation between an individual's number of friends and proximity to their most preferred firm. The following proposition characterizes the effect of this correlation on welfare and pricing.

Proposition 8 *Consider f' and f'' which satisfy assumption 1, then prices, producer profits and welfare will be lower under f' than f''.*

An individual with more friends is more likely to be aware of both firms. When marginally located individuals, people who are located equidistant from either firm

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in our symmetric equilibrium, are more likely to have more friendships, then demand will be more sensitive to price, since these individuals are more likely to know about both firms. In equilibrium, this results in a lower price level. From a welfare perspective, the same mass of individuals make a mistake by choosing a product which is not their most preferred; however, the expected welfare loss under f' is greater than under f'' because the expected distance of these individuals from their preferred firm is larger. The losses associated with marginal individuals choosing the "wrong" firm are relatively small compared (zero for an individual at a distance $x = \frac{1}{2}$) to the losses associated with consumers located close to either one of the firms making the "wrong" choice (t for an individual located a distance $x = 0$ from their preferred firm). Hence, a symmetric correlation which increases the connectivity of marginal individuals while decreasing the connectivity of individuals closer to the firms will tend to reduce welfare because the identity of the individuals making the mistake is now different.

We also find that our metrics of welfare do not move in an intuitive manner under this type of correlation. Awareness is unrelated to any correlation between connectedness and location, whereas demand sensitivity is negatively related to welfare.

Proposition 9 *Consider f' and f'' which satisfy assumption 1, then awareness is the same under f' and f'', and demand sensitivity is lower under f' than f''.*

A correlation between an individual's location and their number of friends does not affect the fraction of individuals who become aware of both firms. However, it does affect where those individuals are likely to be located. A correlation which shifts friendships towards marginal consumers will tend to make demand more sensitive to prices. As we discussed above, this type of change lowers prices but is bad for welfare, since the inefficiency associated with a marginal individual making a mistake is small relative to someone with a preference for one or other of the firms.

The overall effect on consumer welfare depends on the nature of the change to the distribution of friendships. Reallocating friendships from a small number of people to marginal consumers reduces prices for everyone but only induces an inefficiency for a small number of people. The following example illustrates that there is a robust sense that these sorts of changes will tend to improve consumer welfare.

Example of robustness of improving consumer welfare. We consider an example of shifting some friendships from individuals located close to each of the firms to individuals located closer to the marginal consumer. There are two countervailing effects of such a change on consumer welfare. The welfare loss (from choosing the "wrong" firm) to a person located close to one of the firms is greater than the loss for people who are located equidistant from each. This type of inefficiency is increased by shifting friendships away from individuals located close to one of the firms. However, the benefit of increasing the connectivity of marginal consumers

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is that it makes these individuals more sensitive to the prices chosen by the firms. This increases price competition between the firms, resulting in lower prices, which benefit all consumers.

In this section, we consider a shift of friendships from individuals located close to one or another of the firms towards the marginal consumers. We find that all changes (of the kind we consider) improve consumer welfare. Define the following change to the distribution of friendships $q_k(x) = p_k - \varepsilon$; $q_{k-1}(x) = p_{-1} + \varepsilon$ for locations $x \leq \delta$ and $x \geq 1 - \delta$ where δ is small, and $q_k(x) = p_k + \frac{\varepsilon}{\alpha}$; $q_{k-1}(x) = p_{k-1} - \frac{\varepsilon}{\alpha}$ for $\frac{1}{2} - \alpha\delta \leq x \leq \frac{1}{2} + \alpha\delta$, where $0 < \alpha \leq \frac{\frac{1}{2} - \delta}{\delta}$.

Proposition 10 *For all values of α this change improves consumer surplus.*

We see that redistributing friendships towards individuals closer to the marginal consumer improves welfare. Even in the most extreme case in our example, whereby friendships are reduced for individuals in a small neighborhood around both friends and are then redistributed uniformly across the rest of the population, this is sufficient to improve consumer welfare. The reason is that the price change induced by this change is experienced by all consumers. Even though the friendships are evenly distributed across all individuals $\delta \leq x \leq 1 - \delta$ the improvement in the elasticity of the marginal consumers improves prices for consumers sufficiently to offset the welfare associated with higher average travel costs. Finally, note that moving some friendships to individuals closer to the marginal consumer only changes prices if the individuals immediately around the marginal consumer receive some of the friendships. Otherwise, prices are unchanged and consumer welfare becomes unambiguously worse.

6.2   Asymmetric Correlation

In this subsection, we consider a correlation whereby one firm is located closer to individuals with more friends than the other firm. We again model this correlation by placing conditions on the distribution of individuals who are aware of both firms. Without loss of generality, we assume that the firm located close to individuals with more friends is the representative firm. We capture this correlation by assuming that the density $f(x)$ is strictly decreasing in x. The following proposition characterizes the properties of a pricing equilibrium in a market where individuals located closer to one firm are better connected than the other.

Proposition 11 *Firm 1 prices higher than firm 2, $P_1 > P_2$, and obtains a larger market share.*

The firms choose different prices in equilibrium. An equal fraction of individuals find out about each product; however, the distribution of individuals who find out about both are located closer to the representative firm. This results in the

representative firm setting a higher price but nonetheless capturing a greater share of these individuals. The representative firm thus makes a higher profit from being located closer to individuals with more friends. We next consider the welfare effects of this correlation between location and number of friends relative to a network with no correlation.

Proposition 12 *Suppose $\frac{f(x)+f(1-x)}{2} = 1$ and $f(x) > f(1-x)$ for all $x \in \left[0, \frac{1}{2}\right]$, then welfare is lower than in a population where connectedness is independent of location.*

This proposition highlights a source of inefficiency from the asymmetric correlation between connectedness and location. The asymmetry results in individuals closer to the representative firm being more likely to be aware of both firms. It is this asymmetry in prices which results in some consumers, who are aware of both firms, nonetheless choosing the firm further away from themselves because of the lower price. In our model, this corresponds to some individuals located $x < \frac{1}{2}$ amongst the ϕ individuals who are aware of both firms purchasing from firm 2 because of the price difference.

7   Extension to Generalized Number of Firms

Our first step is to characterize the demand curve facing a representative firm $D(P, \{p_k\})$. This is more challenging than the case where everyone is perfectly aware of all products because firms may sell to consumers located far away on the circle. This occurs when these individuals have not found out about any alternative firms located closer to themselves. This means competition is not localized, as in the Salop (1979) model, but is closer to the model of advertising with differentiated products as presented in Grossman and Shapiro (1984). We again assume that we are in the parameter range where a symmetric pure strategy pricing equilibrium exists. We also return to the case where friendships are independent of an individuals location x.

We begin by characterizing the behavior of a representative firm. In doing so, we assume that the remaining firms are playing the symmetric equilibrium with price level $\overline{P}$ and characterize the demand for the firm as a function of its own price. Assume that the circumference of the circle is now 1. The cutoff for the indifferent consumer for whom the representative firm is their m'th preferred firm is:

$$x_m = \frac{\overline{P} - P}{2t} + \frac{m}{2n} \tag{9}$$

Counting consumers on both sides of the representative firm, the mass of consumers

in each group is:

$$D_m = \begin{cases} \frac{\overline{P}-P}{t} + \frac{1}{n} & \text{for } m = 1 \\ \frac{1}{n} & \text{for } 1 < m < n \\ \frac{1}{n} - \frac{\overline{P}-P}{t} & \text{for } m = n \end{cases} \qquad (10)$$

Denote the probability of selling to consumers in each group by ψ_m then

$$\psi_m = \sum_k p_k \left[\frac{n-m+1}{n} \right]^k \left[1 - \left(\frac{n-m}{n-m+1} \right)^k \right] \qquad (11)$$

We can now write demand as

$$\begin{aligned} D &= \sum_{m=1}^{n} D_m \psi_m \qquad &(12) \\ &= \left[\sum_k p_k \left(1 - \left(\frac{n-1}{n} \right)^k - \frac{1}{n^k} \right) \right] \frac{\overline{P}-P}{t} + \frac{1}{n} \end{aligned}$$

It is then straightforward to show that the markup in the symmetric equilibrium is given by

$$\overline{P} = \frac{t}{n} \frac{1}{\sum_k p_k \left(1 - \left(\frac{n-1}{n} \right)^k - \frac{1}{n^k} \right)} \qquad (13)$$

Proposition 13 *The comparative statics in Proposition 2 continue to hold in the case of a generalized number of firms.*

The denominator in equation 13 is increasing and concave in k, as is the denominator in equation 4 for the two-firm case. This is the key property of the pricing relationship which generates the comparative statics in Proposition 2 and continues to hold here. Next we consider the comparative statics of the equilibrium with respect to the number of firms in the market.

Proposition 14 *Prices and profits are decreasing in the number of firms.*

As we might expect, the market becomes increasingly competitive and prices and profits go down, as we increase the number of firms. However, as we see in the next proposition, the network does impose a limit on the extent to which these competitive forces may act.

Proposition 15 *In the limit of a large number of firms, prices are strictly greater than marginal costs, in particular prices approach $c + \frac{t}{E[k]}$.*

Here, even as the number of firms increases, there is a strictly positive lower bound on the markups charged by firms. The social network is the conduit for information and it determines the intensity of price competition between firms. The limit as the number of firms becomes large is the markup that would be achieved in the fully informed model with $E[k]$ symmetric firms located equidistant around the circle.

The final proposition in this section considers the efficiency of free entry into the market. We assume that firms have fixed costs F and that free entry will dissipate profits such that the number of firms that enter the market n^{FE} is the largest integer satisfying:

$$\frac{t}{\left(n^{FE}\right)^2} \frac{1}{\sum_k p_k \left(1 - \left(\frac{n^{FE}-1}{n^{FE}}\right)^k - \frac{1}{(n^{FE})^k}\right)} \geq F \tag{14}$$

Firm entry improves welfare by reducing the average travel cost incurred by consumers. The welfare maximizing number of firms n^{WM} is the largest integer satisfying:

$$t \left[\overline{x}\left(n^{WM}-1\right) - \overline{x}\left(n^{WM}\right)\right] \geq F \tag{15}$$

where $\overline{x}(n)$ is the expected travel cost of a consumer when there are n firms. Our proposition shows that, inefficiently, too many firms enter the market as the fixed costs of entry become small.

Proposition 16 *For small fixed costs, $F < \overline{F}$ free entry results in more firms than the welfare maximizing number of firms, $n^{FE}(F) > n^{WM}(F)$.*

There is excess entry in our model as the number of firms in the market increases. The presence of a "business stealing" effect leading to excessive entry is a common finding in industrial organization models of this kind (see, for instance, Grossman and Shapiro (1984) and Mankiw and Whinston (1986)). We have confirmed here that this property carries over to our model under social learning.

8 Conclusion

We have introduced social learning into a workhorse model of product differentiation. The model is simple, yet permits a variety of comparative statics of network characteristics on outcomes such as pricing and welfare. In our analysis, we find that features of the social network such as homophily and correlations between an individual's valuations and number of friends have non-obvious effects on pricing and welfare. For instance, changing the amount of homophily may improve welfare and have little effect on the price; changing the correlation may reduce prices but nonetheless make welfare worse. These features also highlight some of the challenges of using simple metrics, such as brand awareness surveys and estimates of demand sensitivity, to infer the informational efficiency of markets.

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References

[1] Bagwell, K. (2007). The economic analysis of advertising. *Handbook of Industrial Organization*, 3, 1701-1844.

[2] BenYishay, A. and Mobarak, M., (2015), Communicating with farmers through their social networks. *Working Paper*.

[3] Bergemann, D., & Välimäki, J. (2006). Dynamic pricing of new experience goods. *Journal of Political Economy*, 114(4), 713-743.

[4] Chevalier, J. A., & Mayzlin, D. (2006). The effect of word of mouth on sales: Online book reviews. *Journal of Marketing Research*, 43(3), 345-354.

[5] Chintagunta, P. K., Gopinath, S., & Venkataraman, S. (2010). The effects of online user reviews on movie box office performance: Accounting for sequential rollout and aggregation across local markets. *Marketing Science*, 29(5), 944-957.

[6] Ellison, G., & Ellison, S. F. (2009). Search, obfuscation, and price elasticities on the internet. *Econometrica*, 77(2), 427-452.

[7] Ellison, G., & Wolitzky, A. (2012). A search cost model of obfuscation. *The RAND Journal of Economics*, 43(3), 417-441.

[8] Galeotti, A. (2010). Talking, searching, and pricing. *International Economic Review*, 51(4), 1159-1174.

[9] Galeotti, A., & Goyal, S. (2009). Influencing the influencers: A theory of strategic diffusion. *The RAND Journal of Economics*, 40(3), 509-532.

[10] Galeotti, A. and Mattozzi, A. (2011), "Strategic information transmission in networks," *American Economic Journal: Microeconomics*, 3 (1), 307-27.

[11] Golub, B., & Jackson, M. O. (2012). How homophily affects the speed of learning and best-response dynamics. *The Quarterly Journal of Economics*, 127(3), 1287-1338.

[12] Goyal, S., & Kearns, M. (2012). Competitive contagion in networks. *In Proceedings of the Forty-Fourth Annual ACM symposium on Theory of Computing*, 759-774.

[13] Grossman, S. and Shapiro, C. (1984). Informative advertising with differentiated products. *The Review of Economic Studies*, 51(1), 63-81.

[14] Grubb, M., (2009). Selling to Overconfident Consumers. *American Economic Review*, 99(5), 1770-1805.

[15] Heidhues, P., & Koszegi, B. (2014). Using Information about Naivete to Price Discriminate. Working Paper.

[16] Herr, P. M., Kardes, F. R., & Kim, J. (1991). Effects of word-of-mouth and product-attribute information on persuasion: An accessibility-diagnosticity perspective. *Journal of Consumer Research*, 17(4), 454-462.

[17] Keaveney, S. M. (1995). Customer switching behavior in service industries: an exploratory study. *The Journal of Marketing*, 71-82.

[18] Keller, E., Fay, B., & Berry, J. (2007). Leading the conversation: Influencers' impact on word of mouth and the brand conversation. The Keller Fay Group, *Word of Mouth Marketing Research Symposium*.

[19] Leskovec, J., Adamic, L. A., & Huberman, B. A. (2007). The dynamics of viral marketing. *ACM Transactions on the Web (TWEB)*, 1(1), 5.

[20] Luca, M. (2011). Reviews, reputation, and revenue: The case of Yelp.com (No. 12-016). Harvard Business School.

[21] Mankiw, N. G., & Whinston, M. D. (1986). Free entry and social inefficiency. *The RAND Journal of Economics*, 17(1), 48-58.

[22] McPherson, M., Smith-Lovin, L., & Cook, J. M. (2001). Birds of a Feather: Homophily in Social Networks. *Annual Review of Sociology*, 27, 415-444.

[23] Miller, G. & Mobarak A. M. "Learning about new technologies through social networks: Experimental evidence on non-traditional stoves in rural Bangladesh," *Marketing Science*, forthcoming.

[24] Piccione, M., & Spiegler, R. (2012). Price Competition under Limited Comparability. *The Quarterly Journal of Economics*, 127(1), 1-39.

[25] Rapoport, A. (1948). Cycle distributions in random nets. *The Bulletin of Mathematical Biophysics*, 10(3), 145-157.

[26] Richins, M. L. (1983). Negative word-of-mouth by dissatisfied consumers: A pilot study. *The Journal of Marketing*, 47(1), 68-78.

[27] Salop, S. (1979). Monopolistic competition with outside goods. *The Bell Journal of Economics*, 10(1), 141-156.

[28] Smallwood, D. E., & Conlisk, J. (1979). Product quality in markets where consumers are imperfectly informed. *The Quarterly Journal of Economics*. 93(1),1-23.

[29] Spiegler, R. (2006). The market for quacks. *The Review of Economic Studies*, 73(4), 1113-1131.

[30] Spiegler, R. (2014). Competitive Framing. *American Economic Journal: Microeconomics*, 6(3), 35–58.

[31] Stahl, D. O. (1989). Oligopolistic pricing with sequential consumer search. *The American Economic Review*, 79(4), 700-712.

[32] Sutton, J. (1980). A model of stochastic equilibrium in a quasi-competitive industry. *The Review of Economic Studies*, 47(4), 705-722.

[33] Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world'networks. *Nature*, 393(6684), 440-442.

[34] Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics*, 101(3), 493-511.

9 Appendix

9.1 Proofs of Propositions

9.1.1 Proof of Proposition 1

Proof. The cutoff type is given by:

$$\widehat{x} = \frac{1}{2} + \frac{P_2 - P_1}{2t}$$

whereby all $x \leq \widehat{x}$ purchase from firm 1 and $x \geq \widehat{x}$ purchase from firm 2. Using this to write firm 1's profits

$$\pi_1 = P_1 \left[\phi \int_0^{\widehat{x}} f(x)\, dx + \frac{1-\phi}{2} \right]$$

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$$\left[\phi \int_0^{\widehat{x}^*} f(x)\, dx + \frac{1-\phi}{2} \right] - P_1^* \frac{\phi}{2t} f\left(\frac{1}{2} + \frac{P_2^* - P_1^*}{2t} \right) = 0$$

impose symmetry $P^* = P_1^* = P_2^*$, and assume symmetry of $f(x)$ around $x = \frac{1}{2}$ and that it is differential almost everywhere.

$$\frac{1}{2} - P^* \frac{\phi}{2t} f\left(\frac{1}{2} \right) = 0$$
$$P^* = \frac{t}{\phi f\left(\frac{1}{2} \right)}$$

∎

9.1.2 Proof of Proposition 2

Proof. Both effects come through the change in ϕ:

$$\phi = 1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}$$

The term $\left(\frac{1}{2}\right)^{k-1}$ is decreasing in k and is convex, hence a FOSD change reduces ϕ and a mean preserving spread increases ϕ. The result is then immediate when we note that for a uniform $f(x) = 1$ the equilibrium price level is $P^* = \frac{t}{\phi}$; welfare is determined by the fraction of individuals who choose the product that is furthest away from their location, which is $\frac{1-\phi}{2}$. ∎

9.1.3 Proof of Proposition 3

Proof. In the case of random connections awareness is

$$\phi = 1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}$$

and demand sensitivity is

$$\left| \frac{\partial Q_i}{\partial P_i}(P^*, P^*) \right| = \frac{\phi}{2t} = \frac{1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}}{2t}$$

The the distribution of friendships influences each through the term term $\sum_k p_k \left(\frac{1}{2}\right)^{k-1}$. Thus each move together in this case. Note that welfare

$$W^* = V - t\left[\int_0^1 (2x - 1)\, g^*(x)\, dx + \frac{1}{2} \right]$$

is increasing in $g^*(x)$ for $x < \frac{1}{2}$ and decreasing for $x > \frac{1}{2}$. The result then follows immediately from equation 5 where greater awareness ϕ (and greater sensitivity of demand) increases $g^*(x)$ for $x < \frac{1}{2}$ and decreases it for $x > \frac{1}{2}$. ∎

9.1.4 Proof of Proposition 4

Proof. For a consumer located at x, we denote the probability that a given friend purchased from firm 1 by $\theta_1(x)$; this is

$$\theta_1(x) = \begin{array}{l} \frac{1-\alpha}{2} + \alpha b(x) \text{ for } x \leq \frac{1}{2} - \delta \text{ or } x \geq \frac{1}{2} + \delta \\ \frac{1-\alpha}{2} + \frac{\alpha}{2\delta}\left[(1-\psi)\left(x+\delta-\frac{1}{2}\right) + \psi\left(\frac{1}{2}-x+\delta\right)\right] \text{ for } \frac{1}{2} - \delta \leq x \leq \frac{1}{2} + \delta \end{array}$$

simplifying

$$\theta_1(x) = \begin{array}{l} \frac{1-\alpha}{2} + \alpha\psi \text{ for } x \leq \frac{1}{2} - \delta \\ \frac{1-\alpha}{2} + \alpha(1-\psi) \text{ for } x \geq \frac{1}{2} + \delta \\ \frac{1-\alpha}{2} + \frac{\alpha}{2}\left[1 + (2\psi - 1)\frac{\frac{1}{2}-x}{\delta}\right] \text{ for } \frac{1}{2} - \delta \leq x \leq \frac{1}{2} + \delta \end{array}$$

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The probability $\phi(x, k)$ of a person at x with k friends knowing about both products is

$$\phi(x, k) = 1 - \left[(\theta_1(x))^k + (1 - \theta_1(x))^k\right]$$

hence the probability of a randomly chosen individual at x is

$$\phi(x) = 1 - \sum_k p_k \left[(\theta_1(x))^k + (1 - \theta_1(x))^k\right]$$

and

$$\phi = 1 - \sum_k p_k \left[\int_0^1 (\theta_1(x))^k + (1 - \theta_1(x))^k \, dx\right]$$

Density at $x = \frac{1}{2}$ is given by

$$f\left(\frac{1}{2}\right) = \frac{1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}}{\phi}$$

We can substitute this in to find equilibrium prices:

$$
\begin{aligned}
P^* &= \frac{t}{\phi f\left(\frac{1}{2}\right)} \\
&= \frac{t}{1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}}
\end{aligned}
$$

which is independent of α. Demand for each product is also constant at $\frac{1}{2}$ in the symmetric equilibrium, so producers' profits are unchanged. Consumer welfare is therefore inversely related to the fraction of buyers who purchase the product from the closest firm. This is

$$
\int_0^{\frac{1}{2}} \sum_k p_k \left[1 - (1 - \theta_1(x))^k\right] dx + \int_{\frac{1}{2}}^1 \sum_k p_k \left[1 - (\theta_1(x))^k\right]
$$

$$
= 1 - \int_0^{\frac{1}{2}} \sum_k p_k (1 - \theta_1(x))^k - \int_{\frac{1}{2}}^1 \sum_k p_k (\theta_1(x))^k
$$

We see that consumer welfare is decreasing by noting that $\theta_1(x)$ is increasing in α for $x \leq \frac{1}{2}$ and decreasing for $x \geq \frac{1}{2}$. ∎

9.1.5 Proof of Proposition 5

Proof. Proposition 4 shows that welfare increases in the amount of homophily. Hence it suffices to show that awareness is decreasing in the level of homophily α and demand sensitivity is independent of it. We saw in the previous proof that

$$\phi(x) = 1 - \sum_k p_k \left[(\theta_1(x))^k + (1 - \theta_1(x))^k\right]$$

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and also that $\theta_1(x)$ is increasing in α. Hence awareness is decreasing as homophily is increasing. The sensitivity of demand is given by

$$\frac{\phi f\left(\frac{1}{2}\right)}{2t}$$

and as we saw in the previous proof $\phi f\left(\frac{1}{2}\right) = 1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}$ so is independent of the level of homophily. ∎

9.1.6   Proof of Proposition 6

Proof. In period 2, we can derive the sets of individuals for whom a representative firm is the mth preferred firm. The cutoff types are determined as:

$$x_m = \begin{cases} \frac{1}{3} + \frac{\overline{P}-P}{t} & \text{for } m = 1 \\ \frac{2}{3} + \frac{\overline{P}-P}{t} & \text{for } m = 2 \end{cases}$$

We can write demand as

$$D = 2\left[\int_0^{x_1} g_1(x)\,dx + \int_{x_1}^{x_2} g_2(x)\,dx + \int_{x_2}^1 g_3(x)\,dx\right]$$

where the functions g_m are the probability at a location x that a person buys from their m'th preferred firm. The derivative of demand with respect to price is

$$\frac{dD}{dP} = -\frac{1}{t}\left[g_1(x_1) - g_2(x_1) + g_2(x_2) - g_3(x_2)\right]$$

hence the markup in the symmetric equilibrium is given by

$$P - c = \frac{t}{n}\frac{1}{g_1\left(\frac{1}{3}\right) - g_2\left(\frac{1}{3}\right) + g_2\left(\frac{2}{3}\right) - g_3\left(\frac{2}{3}\right)}$$

now evaluating the terms $g_m(x_m), g_m(x_{m+1})$ ∎

$$g_1\left(\frac{1}{3}\right) = 1 - \sum_k p_k \left[\alpha\left(\frac{1}{2}(1-\psi_1) + \frac{1}{2}(1-\psi_2)\right) + (1-\alpha)\frac{2}{3}\right]^k$$

$$g_2\left(\frac{1}{3}\right) = \sum_k p_k \left[\alpha\left(\frac{1}{2}(1-\psi_1) + \frac{1}{2}(1-\psi_2)\right) + (1-\alpha)\frac{2}{3}\right]^k$$

$$\times \left[1 - \frac{\left[\alpha(\psi_3) + (1-\alpha)\frac{1}{3}\right]^k}{\left[\alpha\left(\frac{1}{2}(1-\psi_1) + \frac{1}{2}(1-\psi_2)\right) + (1-\alpha)\frac{2}{3}\right]^k}\right]$$

$$g_2\left(\frac{2}{3}\right) = \sum_k p_k \left(\alpha(1-\psi_1) + (1-\alpha)\frac{2}{3}\right)^k$$

$$\times \left[1 - \frac{\left(\alpha\left[\frac{1}{2}\psi_2 + \frac{1}{2}\psi_3\right] + (1-\alpha)\frac{1}{3}\right)^k}{\left(\alpha(1-\psi_1) + (1-\alpha)\frac{2}{3}\right)^k}\right]$$

$$g_3\left(\frac{2}{3}\right) = \sum_k p_k \left(\alpha\left(\frac{1}{2}\psi_2 + \frac{1}{2}\psi_3\right) + (1-\alpha)\frac{1}{3}\right)^k$$

Putting this together

$$g_1\left(\frac{1}{3}\right) - g_3\left(\frac{2}{3}\right) - g_2\left(\frac{1}{3}\right) + g_2\left(\frac{2}{3}\right)$$

$$= 1 - \sum_k p_k \left[\alpha\left(1 - \frac{1}{2}(\psi_1 + \psi_2)\right) + (1-\alpha)\frac{2}{3}\right]^k - \sum_k p_k \left(\frac{\alpha}{2}(\psi_2 + \psi_3) + (1-\alpha)\frac{1}{3}\right)^k$$

$$- \sum_k p_k \left[\alpha\left(1 - \frac{1}{2}(\psi_1 + \psi_2)\right) + (1-\alpha)\frac{2}{3}\right]^k + \sum_k p_k \left[\alpha\psi_3 + (1-\alpha)\frac{1}{3}\right]^k$$

$$+ \sum_k p_k \left(\alpha(\psi_2 + \psi_3) + (1-\alpha)\frac{2}{3}\right)^k - \sum_k p_k \left(\frac{\alpha}{2}(\psi_2 + \psi_3) + (1-\alpha)\frac{1}{3}\right)^k$$

simplifying

$$= 1 - 2\sum_k p_k \left[\alpha\left(1 - \frac{1}{2}(\psi_1 + \psi_2)\right) + (1-\alpha)\frac{2}{3}\right]^k - 2\sum_k p_k \left(\frac{\alpha}{2}(\psi_2 + \psi_3) + (1-\alpha)\frac{1}{3}\right)^k$$

$$+ \sum_k p_k \left(\alpha(\psi_2 + \psi_3) + (1-\alpha)\frac{2}{3}\right)^k + \sum_k p_k \left[\alpha\psi_3 + (1-\alpha)\frac{1}{3}\right]^k$$

Now taking the derivative wrt α

$$-2\left(\frac{1}{3}-\frac{1}{2}\left(\psi_1+\psi_2\right)\right)\sum_k kp_k\left[\alpha\left(1-\frac{1}{2}\left(\psi_1+\psi_2\right)\right)+(1-\alpha)\frac{2}{3}\right]^{k-1}$$

$$+2\left(\frac{1}{3}-\frac{1}{2}\left(\psi_2+\psi_3\right)\right)\sum_k kp_k\left(\frac{\alpha}{2}\left(\psi_2+\psi_3\right)+(1-\alpha)\frac{1}{3}\right)^{k-1}$$

$$-2\left(\frac{1}{3}-\frac{1}{2}\left(\psi_2+\psi_3\right)\right)\sum_k kp_k\left(\alpha\left(1-\left(\psi_2+\psi_3\right)\right)+(1-\alpha)\frac{2}{3}\right)^{k-1}$$

$$+2\left(\frac{1}{3}-\frac{1}{2}\left(\psi_1+\psi_2\right)\right)\sum_k kp_k\left[\alpha\psi_3+(1-\alpha)\frac{1}{3}\right]^{k-1}$$

gathering terms and simplifying

$$2\left(\frac{1}{3}-\frac{1}{2}\left(\psi_1+\psi_2\right)\right)\sum_k kp_k\left[\begin{array}{c}\left(\alpha\left(1-\left(\psi_1+\psi_2\right)\right)+(1-\alpha)\frac{1}{3}\right)^{k-1}\\-\left(\alpha\left(1-\frac{1}{2}\left(\psi_1+\psi_2\right)\right)+(1-\alpha)\frac{2}{3}\right)^{k-1}\end{array}\right]$$

$$+2\left(\frac{\psi_1}{2}-\frac{1}{6}\right)\sum_k kp_k\left[\begin{array}{c}\left(\frac{\alpha}{2}\left(1-\psi_1\right)+(1-\alpha)\frac{1}{3}\right)^{k-1}\\-\left(\alpha\psi_1+(1-\alpha)\frac{2}{3}\right)^{k-1}\end{array}\right]$$

substitute the following values for ϕ_m

$$\psi_1 = \frac{5}{9}$$
$$\psi_2 = \frac{3}{9}$$
$$\psi_3 = \frac{1}{9}$$

gives

$$-\frac{2}{9}\sum_k kp_k\left[\left(\alpha\left(\frac{1}{9}\right)+(1-\alpha)\frac{1}{3}\right)^{k-1}-\left(\alpha\frac{5}{9}+(1-\alpha)\frac{2}{3}\right)^{k-1}\right]$$

$$+\frac{2}{9}\sum_k kp_k\left[\left(\alpha\left(\frac{2}{9}\right)+(1-\alpha)\frac{1}{3}\right)^{k-1}-\left(\alpha\frac{5}{9}+(1-\alpha)\frac{2}{3}\right)^{k-1}\right]$$

simplifying

$$\frac{2}{9}\sum_k kp_k\left[\left(\alpha\left(\frac{2}{9}\right)+(1-\alpha)\frac{1}{3}\right)^{k-1}-\left(\alpha\left(\frac{1}{9}\right)+(1-\alpha)\frac{1}{3}\right)^{k-1}\right]>0$$

This is positive, hence prices are decreasing in α. Alternatively, substituting in

$$\psi_1 = 1$$
$$\psi_2 = 0$$
$$\psi_3 = 0$$

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gives:

$$2\left(-\frac{1}{6}\right)\sum_k kp_k\left[\left((1-\alpha)\frac{1}{3}\right)^{k-1}-\left(\alpha\frac{1}{2}+(1-\alpha)\frac{2}{3}\right)^{k-1}\right]$$

$$+2\left(\frac{1}{3}\right)\sum_k kp_k\left[\left((1-\alpha)\frac{1}{3}\right)^{k-1}-\left(\alpha+(1-\alpha)\frac{2}{3}\right)^{k-1}\right]$$

which is negative, hence prices are increasing in α.

9.1.7 Proof of Proposition 7

Proof. It is immediate that

$$\phi^C < \phi^{NC}$$

from equations 6 and 7. Prices are given by

$$P = \frac{t}{\phi}$$

also

$$g^*(x) = \begin{cases} \frac{1+\phi}{2} & \text{if } x < \frac{1}{2} \\ \frac{1-\phi}{2} & \text{if } x > \frac{1}{2} \end{cases}$$

hence it is also immediate that prices are lower and welfare greater without clustering. Finally demand sensitivity is $\frac{\phi}{2t}$ which is greater without clustering than with. ∎

9.1.8 Proof of Proposition 8

Proof. Prices are given by $P = \frac{1}{\phi f\left(\frac{1}{2}\right)}$. By assumption, $f'\left(\frac{1}{2}\right) > f''\left(\frac{1}{2}\right)$ and note that ϕ is the same under f' and f'' hence prices are lower under f'; producers always sell to $\frac{1}{2}$ mass of consumers in the symmetric equilibrium, so profits also decrease. Welfare is determined by the welfare loss from individuals buying from the firm located further away. This is given by

$$-2\int_0^{\frac{1}{2}} \frac{(1-\phi f(x))}{2}2\left(\frac{1}{2}-x\right)dx$$

$$= -2\int_0^{\frac{1}{2}} (1-\phi f(x))\left(\frac{1}{2}-x\right)dx$$

$$= -\left[\frac{1-\phi}{2}\right] + 2\int_0^{\frac{1}{2}} x\,dx - 2\phi\int_0^{\frac{1}{2}} xf(x)\,dx$$

where $\int_0^{\frac{1}{2}} xf(x)\,dx$ is larger for f' than f'' hence the welfare loss is greater. ∎

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9.1.9 Proof of Proposition 9

Proof. It is immediate that awareness ϕ is independent of the correlation structure. Demand sensitivity is given by

$$\frac{\phi f\left(\frac{1}{2}\right)}{2t}$$

Now f' is first order stochastically dominated by f'' and $\int_0^{\frac{1}{2}} f'(x)\,dx = \int_0^{\frac{1}{2}} f''(x)\,dx$ hence $f''\left(\frac{1}{2}\right) < f'\left(\frac{1}{2}\right)$. Therefore demand sensitivity is higher under f' than f'' ∎

9.1.10 Proof of Proposition 10

Proof. At the locations $x : x \le \delta$ and $x \ge 1 - \delta$ the the probability that someone only finds out about the "wrong" firm changes from $\left(\frac{1}{2}\right)^k$ to $\left(\frac{1}{2}\right)^{k-1}$ for the affected individuals. This is a welfare loss for small values of δ of $t\left(\frac{1}{2}\right)^k\left(1 - \frac{\delta}{2}\right)$ across the $2\varepsilon\delta$ individuals affected. The improvement of the match for the people located $\frac{1}{2} - \alpha\delta \le x \le \frac{1}{2} + \alpha\delta$ is $\left(\frac{1}{2}\right)^k \alpha\delta t$ across the $2\varepsilon\delta$ people affected. The net inefficiency of the matching of consumers to firms is:

$$-\left(\frac{1}{2}\right)^k \left(1 - \delta\left(\frac{1}{2} + \alpha\right)\right)$$

The price effect occurs through the change to $P^* = \frac{t}{1-\sum_k p_k\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{k-1}}$. Now, taking the derivative with respect to ε

$$\frac{dP^*}{d\varepsilon} = -\frac{t}{\alpha}\left(\frac{1}{2}\right)^k \left(\frac{1}{1-\sum_k p_k\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{k-1}}\right)^2$$

so the benefits to consumers are approximately

$$\frac{\varepsilon t}{\alpha}\left(\frac{1}{2}\right)^k \left(\frac{1}{1-\sum_k p_k\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{k-1}}\right)^2$$

these outweigh the inefficiency provided

$$\frac{\varepsilon t}{\alpha}\left(\frac{1}{2}\right)^k \left(\frac{1}{1-\sum_k p_k\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{k-1}}\right)^2 > 2\varepsilon\delta t\left(\frac{1}{2}\right)^k\left(1 - \delta\left(\frac{1}{2} + \alpha\right)\right)$$

$$\frac{1}{\alpha}\left(\frac{1}{1-\sum_k p_k\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{k-1}}\right)^2 > 2\delta\left(1 - \delta\left(\frac{1}{2} + \alpha\right)\right)$$

the term $\left(\frac{1}{1-\sum_k p_k\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)^{k-1}}\right)^2 > 1$ and $\left(1 - \delta\left(\frac{1}{2} + \alpha\right)\right) < 1$ so provided

$$\frac{1}{\alpha} > 2\delta$$

which is true, since the upper bound for α is $\frac{\frac{1}{2}-\delta}{\delta}$. ■

9.1.11 Proof of Proposition 11

Proof. The cutoff type is given by:

$$\widehat{x} = \frac{1}{2} + \frac{P_2 - P_1}{2t}$$

whereby all $x \leq \widehat{x}$ purchase from firm 1 and $x \geq \widehat{x}$ purchase from firm 2. Using this to write firm 1's profits:

$$\pi_1 = P_1 \left[\phi \int_0^{\widehat{x}} f(x) \, dx + \frac{1-\phi}{2} \right]$$

first order condition:

$$\left[\phi \int_0^{\widehat{x}^*} f(x) \, dx + \frac{1-\phi}{2} \right] - P_1^* \frac{\phi}{2t} f \left(\frac{1}{2} + \frac{P_2^* - P_1^*}{2t} \right) = 0$$

unlike earlier, $f(x)$ is not symmetric around $\frac{1}{2}$. We have assumed that $f(x)$ is decreasing and $f\left(\frac{1}{2}\right) = 1$.

$$\phi \left[F(\widehat{x}^*) - \frac{1}{2} - \frac{P_1^*}{2t} f(\widehat{x}^*) \right] + \frac{1}{2} = 0$$
$$\frac{1}{2} + \frac{P_1^*}{2t} f(\widehat{x}^*) - F(\widehat{x}^*) = \frac{1}{2\phi}$$

Similarly, we can find for firm 2

$$\frac{1}{2} + \frac{P_2^*}{2t} f(\widehat{x}^*) - (1 - F(\widehat{x}^*)) = \frac{1}{2\phi}$$

hence

$$\frac{P_2^* - P_1^*}{2t} f(\widehat{x}^*) = 1 - 2F(\widehat{x}^*)$$
$$\left(\widehat{x}^* - \frac{1}{2} \right) f(\widehat{x}^*) = 1 - 2F(\widehat{x}^*)$$

if $\widehat{x}^* > \frac{1}{2}$ the left-hand side is positive and the right-hand side is negative, hence $\widehat{x}^* < \frac{1}{2}$. This implies that $P_1^* > P_2^*$ and $F(\widehat{x}^*) > \frac{1}{2}$. Hence firm 1's market share is $\frac{1-\phi}{2} + \phi F(\widehat{x}^*) > \frac{1}{2}$. There is an inefficiency due to the asymmetric pricing amongst the elastic consumers because some consumers with $x < \frac{1}{2}$ incur the additional travel costs to purchase from firm 2. ■

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9.1.12 Proof of Proposition 12

Proof. The probability of purchasing from the representative firm is:

$$
g\left(x\right) = \begin{cases} \phi f\left(x\right) + \frac{1-\phi f\left(x\right)}{2} & \text{for } x < \widehat{x} \\ \frac{1-\phi f\left(x\right)}{2} & \text{for } x > \widehat{x} \end{cases}
$$

where $\widehat{x} = \frac{1}{2} + \frac{P_2 - P_1}{2t}$ is the location of the marginal consumers. Let g^A and g^I be the probabilities for the asymmetric correlation and independent networks respectively. The difference in welfare between the two is:

$$
\begin{aligned}
W^I - W^A &= t\left[\int_0^1 (2x-1)\left[g^A\left(x\right) - g^I\left(x\right)\right] dx\right] \\
&= t\left[\int_{\widehat{x}^A}^{\frac{1}{2}} (2x-1)\,\phi\left[f\left(x\right) - 1\right] dx\right] > 0
\end{aligned}
$$

where $\widehat{x}^A$ is the location of the marginal consumer in the asymmetric case. ∎

9.1.13 Proof of Proposition 13

Proof. This follows immediately after noting that $1 - \left(\frac{n-1}{n}\right)^k - \frac{1}{n^k}$ is an increasing and concave function of k. ∎

9.1.14 Proof of Proposition 14

Proof. We show that prices are decreasing in the number of firms $\frac{d\overline{P}}{dn} < 0$. This then immediately implies that profits will also be decreasing. Prices are given by

$$
\overline{P} = \frac{t}{n}\frac{1}{\sum_k p_k \left(1 - \left(\frac{n-1}{n}\right)^k - \frac{1}{n^k}\right)}
$$

hence

$$
\begin{aligned}
sign\frac{d\overline{P}}{dn} &= -sign\frac{\partial}{\partial n} n\left[1 - \left(\frac{n-1}{n}\right)^k - \frac{1}{n^k}\right] \\
&= -sign\left\{ \begin{array}{c} \left[1 - \left(\frac{n-1}{n}\right)^k - \frac{1}{n^k}\right] \\ +n\left[\frac{1}{n^2}k\left[\frac{1}{n}\right]^{k-1} - k\frac{1}{n^2}\left[\frac{n-1}{n}\right]^{k-1}\right] \end{array} \right\} \\
&= -sign\left\{ n^k - (n-1)^k - k(n-1)^{k-1} + k - 1 \right\} \\
&= -sign\left\{ \left(\frac{n}{n-1}\right)^k - 1 - \frac{k}{n-1} + \frac{k-1}{(n-1)^k} \right\}
\end{aligned}
$$

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the term inside the *sign* function is positive at $n = 2, k = 3$. We may effectively ignore the fourth term after noting that it is always strictly positive for $k \geq 2$. Note

$$\lim_{n \to \infty} \left(\frac{n}{n-1} \right)^k - 1 - \frac{k}{n-1} = 0$$

We conclude by observing that the expression monotonically decreases and is thus always positive.

$$\frac{\partial}{\partial n} \left[\left(\frac{n}{n-1} \right)^k - 1 - \frac{k}{n-1} \right]$$

$$= \frac{-1}{(n-1)^2} k \left(\frac{n}{n-1} \right)^{k-1} + \frac{k}{(n-1)^2}$$

$$= \frac{k}{(n-1)^2} \left[1 - \left(\frac{n}{n-1} \right)^{k-1} \right] < 0$$

hence

$$\left(\frac{n}{n-1} \right)^k - 1 - \frac{k}{n-1} > 0 \text{ for all } n \geq 2, k \geq 2$$

∎

9.1.15 Proof of Proposition 15

Proof. We have

$$\lim_{n \to \infty} \overline{P} = \lim_{n \to \infty} c + \frac{t}{n} \frac{1}{\sum_k p_k \left(1 - \left(\frac{n-1}{n} \right)^k - \frac{1}{n^k} \right)}$$

$$= c + \frac{t}{\lim_{n \to \infty} n \sum_k p_k \left(1 - \left(\frac{n-1}{n} \right)^k - \frac{1}{n^k} \right)}$$

evaluating

$$\lim_{n \to \infty} n \sum_k p_k \left(1 - \left(\frac{n-1}{n} \right)^k - \frac{1}{n^k} \right)$$

by L'Hopital's rule

$$\lim_{n \to \infty} \frac{\sum_k p_k \left(\frac{k}{n^{k-1}} - k \frac{1}{n^2} \left(\frac{n-1}{n} \right)^{k-1} \right)}{-\frac{1}{n^2}}$$

$$= \sum_k p_k k = E[k]$$

∎

9.1.16 Proof of Proposition 16

Proof. Social welfare improves with more firms because it reduces the distance between the representative firm and its consumers. The average distance of the representative firm to its customers is given by:

$$\overline{x}\left(n\right) = \sum_{m=1}^{n} \overline{x}_m \psi_m$$

where $\overline{x}_m$ is the average distance traveled by customers for whom the representative firm is their mth preferred firm. This is given by:

$$\overline{x}_m = \frac{2m-1}{4n}$$

hence

$$
\begin{aligned}
\overline{x}\left(n\right) &= \sum_{m=1}^{n} \overline{x}_m \psi_m = \sum_{m=1}^{n} \frac{2m-1}{4n} \left(\sum_k p_k \left[\frac{n-m+1}{n}\right]^k \left[1 - \left(\frac{n-m}{n-m+1}\right)^k\right] \right) \\
&= \sum_k p_k \frac{1}{4n^{k+1}} \sum_{m=1}^{n} (2m-1)\left[(n-m+1)^k - (n-m)^k\right] \\
&= \sum_k p_k \frac{1}{4n^{k+1}} \left[\sum_{m=1}^{n} 2m\left[(n-m+1)^k - (n-m)^k\right] - n^k\right] \\
&= \sum_k p_k \frac{1}{4n^{k+1}} \left[\sum_{m=1}^{n} 2m^k - n^k\right] \\
&= \sum_k p_k \frac{2\sum_{m=1}^{n}\left(\frac{m}{n}\right)^k - 1}{4n}
\end{aligned}
$$

Free entry and welfare maximization find the largest integers satisfying:

$$\frac{t}{\left(n^{FE}\right)^2} \frac{1}{\sum_k p_k \left(1 - \left(\frac{n^{FE}-1}{n^{FE}}\right)^k - \frac{1}{\left(n^{FE}\right)^k}\right)} \geq F$$

and

$$t\left[\sum_k p_k \frac{2\sum_{m=1}^{n^{MW}-1} m^k - \left(n^{MW}-1\right)^k}{4\left(n^{MW}-1\right)^{k+1}} - \sum_k p_k \frac{2\sum_{m=1}^{n^{MW}} m^k - \left(n^{MW}\right)^k}{4\left(n^{WM}\right)^{k+1}}\right] \geq F$$

Note for $F \to 0$ both $n^{FE}, n^{MW} \to \infty$. We prove the proposition by showing that for large values of n, the left-hand side of the expression for n^{FE} is larger than the

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expression for n^{NW}. Thus, the value of n that satisfies the latter equation is less than the former. Initially we simplify the expression for n^{NW} :

$$t\left[\sum_k p_k \frac{2\sum_{m=1}^{n^{MW}-1} m^k - \left(n^{MW}-1\right)^k}{4\left(n^{MW}-1\right)^{k+1}} - \sum_k p_k \frac{2\sum_{m=1}^{n^{MW}} m^k - \left(n^{MW}\right)^k}{4\left(n^{WM}\right)^{k+1}}\right]$$

$$\frac{t}{2n}\left[\sum_k p_k\left[\sum_{m=1}^n \left(\frac{m}{n}\right)^k\right] - \frac{n}{n+1}\sum_k p_k\left[\sum_{m=1}^{n+1}\left(\frac{m}{n+1}\right)^k\right] - \frac{1}{2}\left[\frac{1}{n+1}\right]\right]$$

$$\frac{t}{2n}\left[\sum_k p_k\left[\left[1 - \frac{n^{k+1}}{(n+1)^{k+1}}\right]\sum_{m=1}^n \left(\frac{m}{n}\right)^k\right] - \frac{n+\frac{1}{2}}{n+1}\right]$$

We remove the common factor $\frac{t}{n}$ from each expression:

$$\frac{1}{2}\left[\sum_k p_k\left[\left[1 - \frac{n^{k+1}}{(n+1)^{k+1}}\right]\sum_{m=1}^n \left(\frac{m}{n}\right)^k\right] - \frac{n+\frac{1}{2}}{n+1}\right] \leq \frac{1}{n\sum_k p_k\left(1 - \left(\frac{n-1}{n}\right)^k - \frac{1}{n^k}\right)}$$

and compare the limit of large $n \to \infty$ of the resultant terms, we find, that each term approaches different quantities:

$$\lim_{n\to\infty} \frac{1}{2}\left[\sum_k p_k\left[\left[1 - \frac{n^{k+1}}{(n+1)^{k+1}}\right]\sum_{m=1}^n \left(\frac{m}{n}\right)^k\right] - \frac{n+\frac{1}{2}}{n+1}\right] = 0$$

$$\lim_{n\to\infty} \frac{1}{n\sum_k p_k\left(1 - \left(\frac{n-1}{n}\right)^k - \frac{1}{n^k}\right)} = \frac{1}{E[k]}$$

and we conclude that, for sufficiently small F, $n^{FE}(F) > n^{WM}(F)$. ∎

9.2 Existence of symmetric equilibrium in prices

From the proof of Proposition 1, we can find the second order condition; for firm 1, it is given by

$$\partial\left[\left[\phi\int_0^{\widehat{x}^*} f(x)\,dx + \frac{1-\phi}{2}\right] - P_1^*\frac{\phi}{2t}f\left(\frac{1}{2} + \frac{P_2^* - P_1^*}{2t}\right)\right]/\partial P_1$$

$$= -2\frac{\phi}{2t}f\left(\frac{1}{2}\right) + P_1^*\frac{\phi}{(2t)^2}\frac{df\left(\frac{1}{2}\right)}{dx}$$

when f is symmetric and smooth at $\widehat{x}^* = \frac{1}{2}$, $\frac{df\left(\frac{1}{2}\right)}{dx} = 0$, the second order is equal to $-2\frac{\phi}{2t}f\left(\frac{1}{2}\right)$. This ensures we are at a local maximum.. The condition that f is smooth at $\widehat{x}^* = \frac{1}{2}$ is guaranteed in the case of a uniform and the case of homophily. In the symmetric correlation case, it requires that $\{p_k(x)\}$ is smooth in x. The

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symmetry of the problem ensures that if the second order condition holds for firm 1, then it is also satisfied for firm 2.

We show that there are parameters such that a symmetric pure strategy equilibrium exists for the case where f is a uniform distribution. We then note that profits are continuous in f, hence for f sufficiently close to a uniform distribution, and satisfying our symmetry and smoothness assumptions, then a symmetric pure strategy equilibrium will continue to exist.

First we will assume

$$\frac{1}{\phi} + 1 < \frac{V - t}{t} \tag{16}$$

, where we note that $\phi = 1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1}$ and hence that this condition implies

$$1 - \sum_k p_k \left(\frac{1}{2}\right)^{k-1} > \frac{t}{V - 2t}$$

We may always satisfy this condition with a sufficiently connected network.

The profit when f is a uniform distribution is

$$\pi_1\left(P_1, \frac{t}{\phi}\right) = \begin{cases} P_1 \frac{1+\phi}{2} \text{ if } P_1 \leq t\left(\frac{1}{\phi} - 1\right) \\ P_1 \left[\phi\left[\frac{1}{2} + \frac{\frac{t}{\phi} - P_1}{2t}\right] + \frac{1-\phi}{2}\right] \text{ if } t\left(\frac{1}{\phi} - 1\right) \leq P_1 \leq t\left(\frac{1}{\phi} + 1\right) \\ P_1 \frac{1-\phi}{2} \text{ if } t\left(\frac{1}{\phi} + 1\right) \leq P_1 \leq V - t \\ P_1 \frac{1-\phi}{2}\left(\frac{V - P_1}{t}\right) \text{ if } P_1 \geq V - t \end{cases}$$

There are two cases of prices $P_1 \geq t\left(\frac{1}{\phi} + 1\right)$; $P_1 = V - t$ or $P_1 = \frac{V}{2}$, other than $P_1 = \frac{t}{\phi}$, which may be optimal. The conditions for neither to be optimal are:

$$P_1 = \begin{cases} V - t \text{ if } \frac{V}{2} < V - t \\ \frac{V}{2} \text{ if } \frac{V}{2} \geq V - t \end{cases}$$

In each case, profits are:

$$\pi_1 = \begin{cases} (V - t)\frac{1-\phi}{2} \text{ if } \frac{V}{2} < V - t \\ \frac{V^2}{4}\frac{1-\phi}{2t} \text{ if } \frac{V}{2} \geq V - t \end{cases}$$

These are not better than the symmetric equilibrium profits provided

$$(V - t)\frac{1-\phi}{2} \leq \frac{t}{\phi}\frac{1}{2} \text{ if } \frac{V}{2} < V - t$$
$$\frac{V^2}{4}\frac{1-\phi}{2t} \leq \frac{t}{\phi}\frac{1}{2} \text{ if } \frac{V}{2} \geq V - t$$

Rearranging gives

$$\frac{V-t}{t} \leq \frac{1}{\phi(1-\phi)} \text{ if } \frac{V}{2} < V - t$$
$$\frac{V^2}{4t^2} \leq \frac{1}{\phi(1-\phi)} \text{ if } \frac{V}{2} \geq V - t \tag{17}$$

Combining conditions in equations 16 and 17, the following conditions are sufficient:

$$\frac{1}{\phi} + 1 \le \frac{V - t}{t} \le \frac{1}{\phi\left(1 - \phi\right)}$$

and

$$t \le \frac{V}{2}$$

for instance, $\phi = \frac{1}{2}, t = \frac{V}{4.5}$ satisfies the above conditions.

We now note that profits are continuous in f, hence for f sufficiently close to a uniform distribution, and satisfying our symmetry and smoothness assumptions, then a symmetric pure strategy equilibrium will continue to exist for the cases of homophily and symmetric correlation for f close enough to uniform.